

Mathematical studies
Standard level
Paper 1

Thursday 10 November 2016 (afternoon)

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.



Please **do not** write on this page.
Answers written on this page will not
be marked.



20EP02

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Let $p = \frac{\cos x + \sin y}{\sqrt{w^2 - z}}$,

where $x = 36^\circ$, $y = 18^\circ$, $w = 29$ and $z = 21.8$.

- (a) Calculate the value of p . Write down your full calculator display. [2]
- (b) Write your answer to part (a)
 - (i) correct to two decimal places;
 - (ii) correct to three significant figures. [2]
- (c) Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [2]

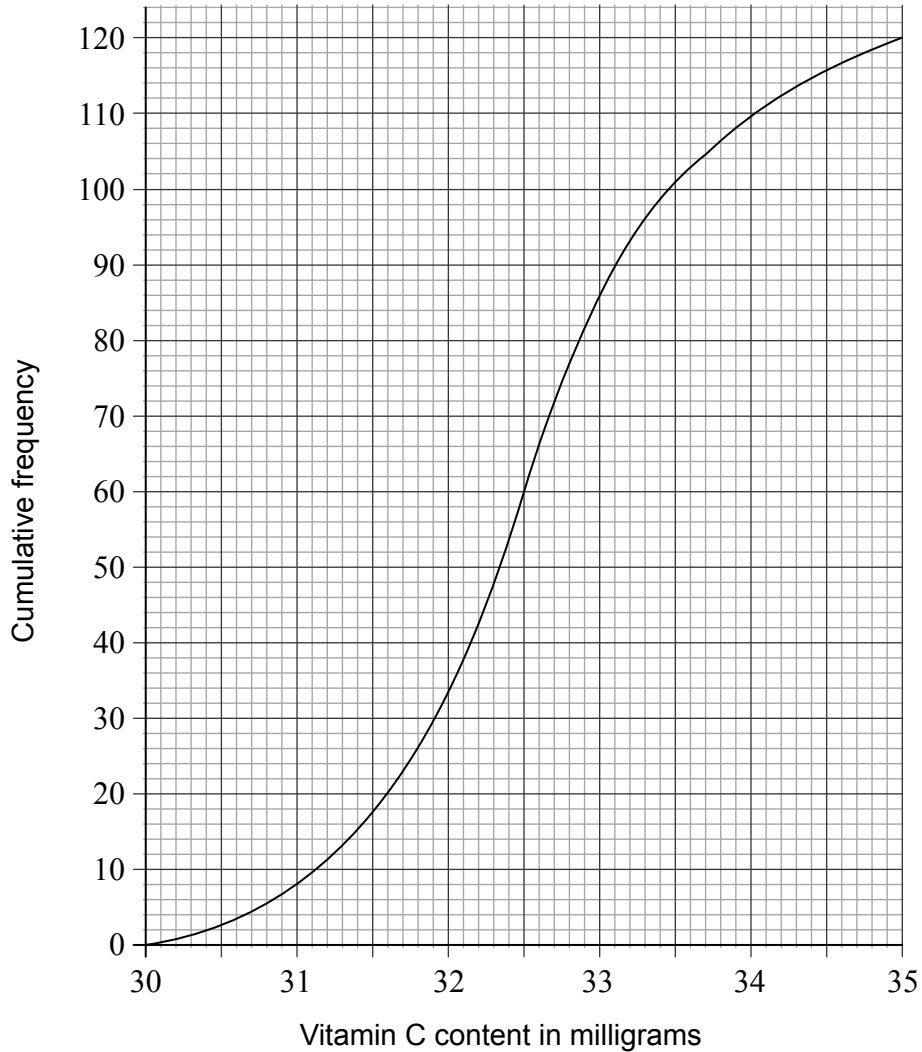
Working:

Answers:

- (a)
- (b) (i)
- (ii)
- (c)



2. A sample of 120 oranges was tested for Vitamin C content. The cumulative frequency curve below represents the Vitamin C content, in milligrams, of these oranges.



- (a) Giving your answer to one decimal place, write down the value of
- (i) the median level of Vitamin C content of the oranges in the sample;
 - (ii) the lower quartile;
 - (iii) the upper quartile.

[3]

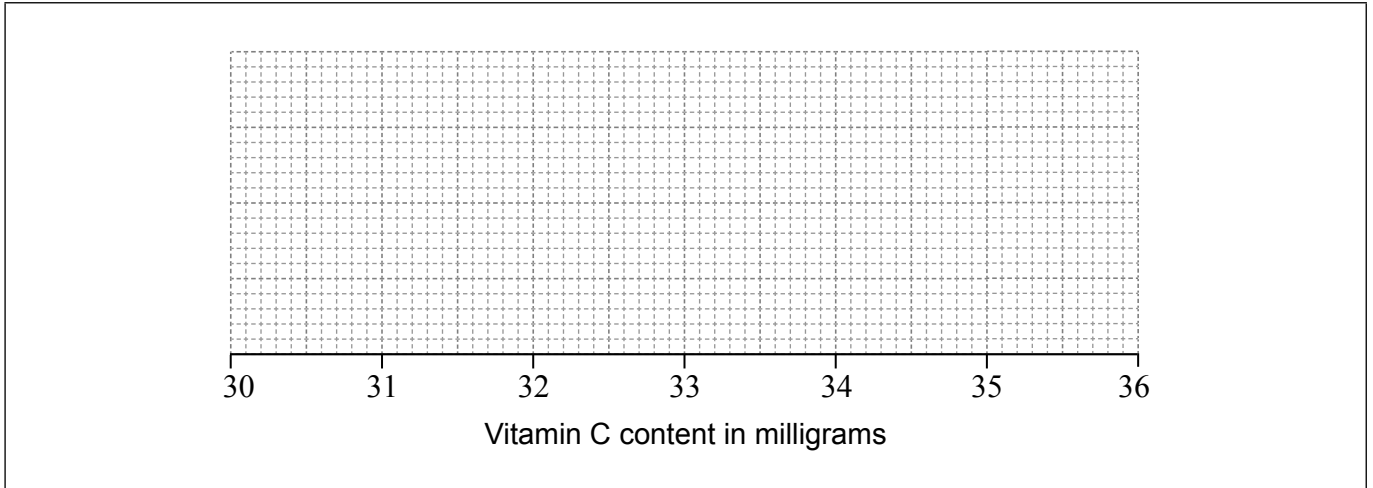
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(Question 2 continued)

The minimum level of Vitamin C content of an orange in the sample was 30.1 milligrams.
The maximum level of Vitamin C content of an orange in the sample was 35.0 milligrams.

- (b) Draw a box-and-whisker diagram on the grid below to represent the Vitamin C content, in milligrams, for this sample. [3]



Working:

Answers:

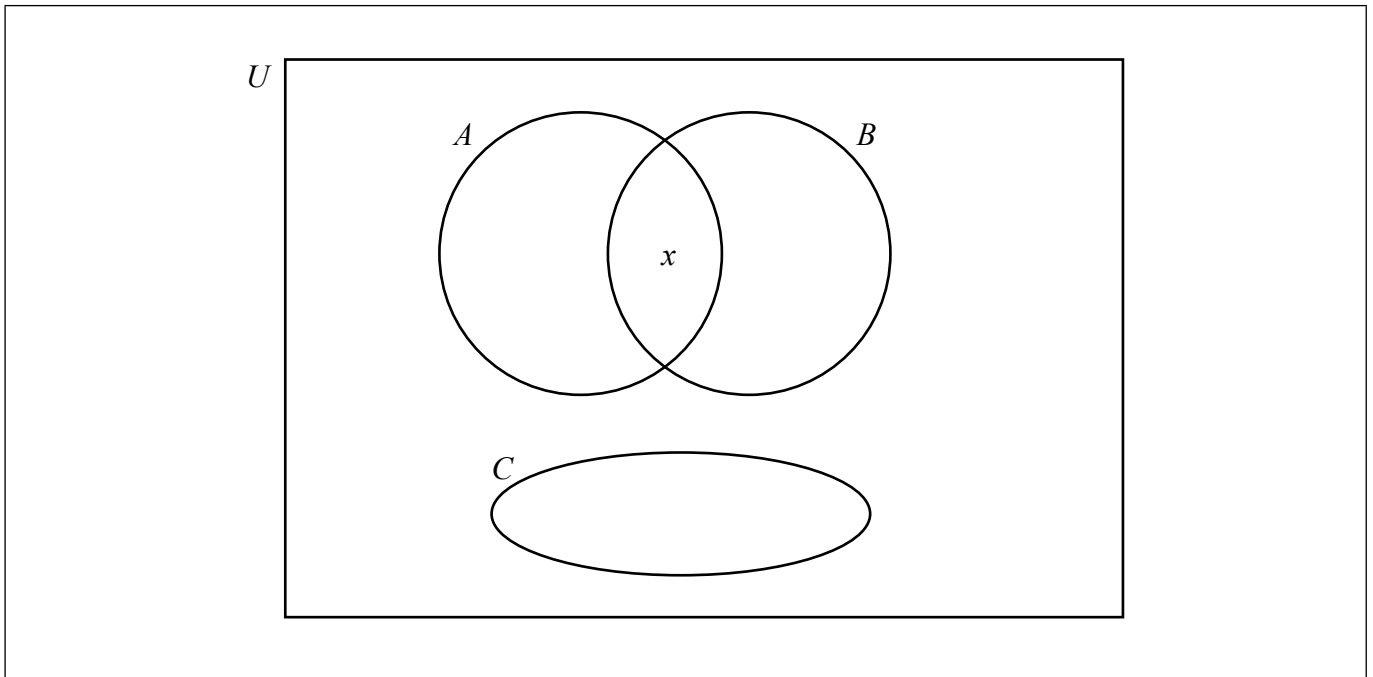
- (a) (i)
- (ii)
- (iii)



20EP05

Turn over

3. The following Venn diagram shows the sets A , B , C and U .
 x is an element of U .



- (a) In the table indicate whether the given statements are True or False. [5]

| Statement | True or False |
|---------------------------|---------------|
| $x \in C$ | |
| $x \subset B$ | |
| $A \cup B \neq \emptyset$ | |
| $A \cap B \subset C$ | |
| $A \cap C = \emptyset$ | |

- (b) On the Venn diagram, shade the region $A \cap (B \cup C)'$. [1]



4. In this question give all answers correct to two decimal places.

Javier takes 5000 US dollars (USD) on a business trip to Venezuela. He exchanges 3000 USD into Venezuelan bolívars (VEF).

The exchange rate is 1 USD = 6.3021 VEF.

- (a) Calculate the amount of VEF that Javier receives. [2]

During his time in Venezuela, Javier spends 1250 USD and 12 000 VEF. On his return home, Javier exchanges his remaining VEF into USD.

The exchange rate is 1 USD = 8.7268 VEF.

- (b) Calculate the total amount, in USD, that Javier has remaining from his 5000 USD after his trip to Venezuela. [4]

Working:

Answers:

- (a)
- (b)



5. Consider the following logic propositions.

p : Sandi gets up before eight o'clock

q : Sandi goes for a run

r : Sandi goes for a swim

(a) Write down in words the compound proposition

$$p \Rightarrow (q \vee r).$$

[3]

(b) Complete the following truth table.

[2]

| p | q | r | $q \vee r$ | $p \Rightarrow (q \vee r)$ |
|-----|-----|-----|------------|----------------------------|
| T | T | T | | |
| T | T | F | | |
| T | F | T | | |
| T | F | F | | |
| F | T | T | | |
| F | T | F | | |
| F | F | T | | |
| F | F | F | | |

(c) On a morning when Sandi does **not** get up before eight o'clock, use your truth table to determine whether $p \Rightarrow (q \vee r)$ is a tautology, contradiction or neither.

[1]

Working:

Answers:

- (a)
-
-
- (c)



6. A hospital collected data from 1000 patients in four hospital wards to review the quality of its healthcare. The data, showing the number of patients who became infected during their stay in hospital, was recorded in the following table.

| | Ward | | | | Total |
|-----------------------|---------|-------------|--------|---------|-------|
| | Pasteur | Nightingale | Jenner | Fleming | |
| Patients infected | 44 | 27 | 13 | 16 | 100 |
| Patients not infected | 281 | 303 | 182 | 134 | 900 |
| Total | 325 | 330 | 195 | 150 | 1000 |

A χ^2 -test was performed at the 5% significance level.
The critical value for this test is 7.815.

The null hypothesis for the test is

H_0 : Becoming infected during a stay in the hospital is independent of the ward.

- (a) Find the expected frequency of the patients who became infected whilst in Nightingale ward. [2]
- (b) For this test, write down the χ^2 statistic. [2]
- (c) State, giving a reason, whether the null hypothesis should be rejected. [2]

Working:

Answers:

- (a)
- (b)
- (c)



7. A balloon in the shape of a sphere is filled with helium until the radius is 6 cm.

(a) Calculate the volume of the balloon.

[2]

The volume of the balloon is increased by 40%.

(b) Calculate the radius of the balloon following this increase.

[4]

Working:

Answers:

(a)

(b)



8. Passengers of Flyaway Airlines can purchase tickets for either Business Class or Economy Class.

On one particular flight there were 154 passengers.

Let x be the number of Business Class passengers and y be the number of Economy Class passengers on this flight.

(a) Use the above information to write down an equation in x and y . [1]

On this flight, the cost of a ticket for each Business Class passenger was 320 euros and the cost of a ticket for each Economy Class passenger was 85 euros. The total amount that Flyaway Airlines received for these tickets was 14970 euros.

(b) Use the information about the cost of tickets to write down a second equation in x and y . [1]

(c) Find the value of x and the value of y . [2]

The airline's finance officer wrote down the total amount received by the airline for these tickets as 14270 euros.

(d) Find the percentage error. [2]

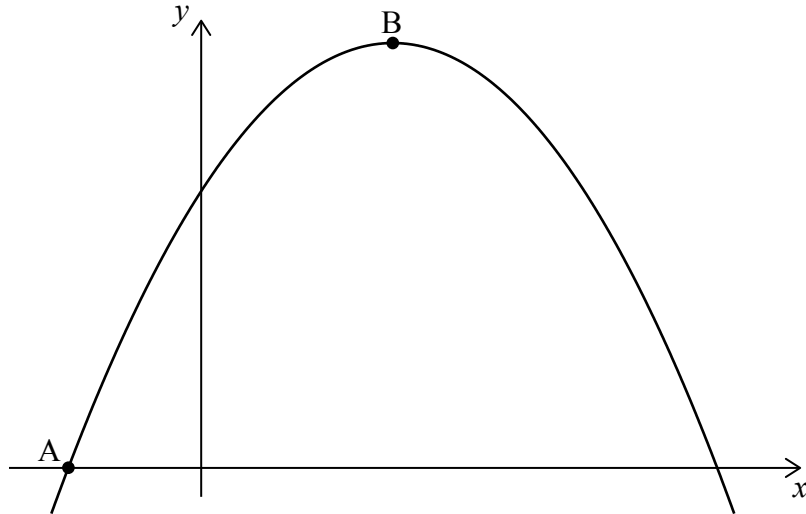
Working:

Answers:

- (a)
- (b)
- (c)
- (d)



9. The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the x -axis at the point $A(-1, 0)$ and has its vertex at the point $B(3, 16)$.

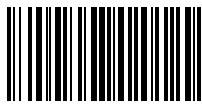


- (a) Write down the equation of the axis of symmetry for this graph. [2]
- (b) Find the value of b . [2]
- (c) Write down the range of $f(x)$. [2]

Working:

Answers:

- (a)
- (b)
- (c)



10. A hydraulic hammer drives a metal post vertically into the ground by striking the top of the post. The distance that the post is driven into the ground, by the n th strike of the hammer, is d_n .

The distances $d_1, d_2, d_3, \dots, d_n$ form a geometric sequence.

The distance that the post is driven into the ground by the first strike of the hammer, d_1 , is 64 cm.

The distance that the post is driven into the ground by the second strike of the hammer, d_2 , is 48 cm.

- (a) Find the value of the common ratio for this sequence. [2]
- (b) Find the distance that the post is driven into the ground by the eighth strike of the hammer. [2]
- (c) Find the **total depth** that the post has been driven into the ground after 10 strikes of the hammer. [2]

Working:

Answers:

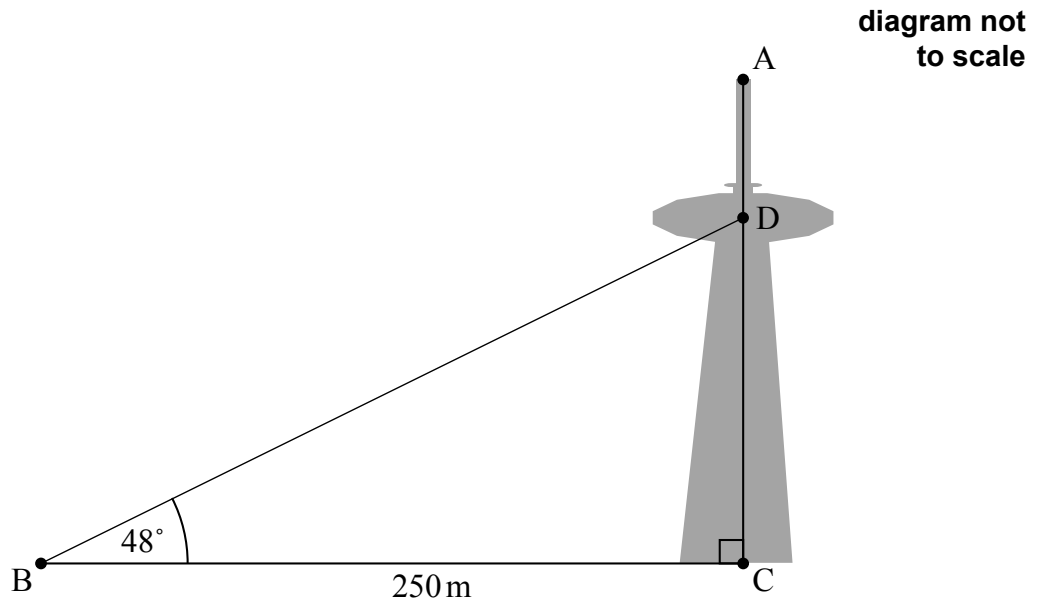
(a)

(b)

(c)



11. AC is a vertical communications tower with its base at C. The tower has an observation deck, D, three quarters of the way to the top of the tower, A.



From a point B, on horizontal ground 250 m from C, the angle of elevation of D is 48° .

- (a) Calculate CD, the height of the observation deck above the ground. [2]
- (b) Calculate the angle of depression from A to B. [4]

Working:

Answers:

- (a)
- (b)



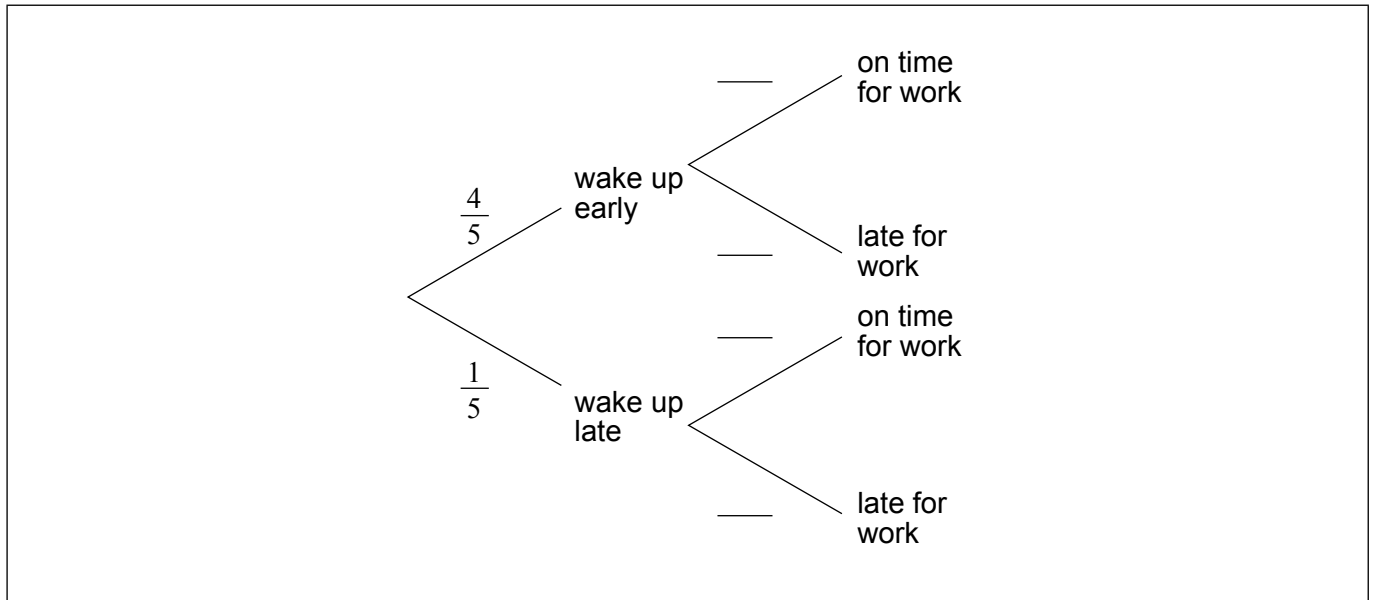
12. On a work day, the probability that Mr Van Winkel wakes up early is $\frac{4}{5}$.

If he wakes up early, the probability that he is on time for work is p .

If he wakes up late, the probability that he is on time for work is $\frac{1}{4}$.

(a) Complete the tree diagram below.

[2]



The probability that Mr Van Winkel arrives on time for work is $\frac{3}{5}$.

(b) Find the value of p .

[4]

Working:

Answers:

(b)



13. A comet orbits the Sun and is seen from Earth every 37 years. The comet was first seen from Earth in the year 1064.

(a) Find the year in which the comet was seen from Earth for the fifth time. [3]

(b) Determine how many times the comet has been seen from Earth up to the year 2014. [3]

Working:

Answers:

(a)

(b)



14. The equation of a curve is $y = \frac{1}{2}x^4 - \frac{3}{2}x^2 + 7$.

(a) Find $\frac{dy}{dx}$. [2]

The gradient of the tangent to the curve at a point P is -10.

(b) Find the coordinates of P. [4]

Working:

Answers:

(a)

(b)



15. Gabriella purchases a new car.

The car's value in dollars, V , is modelled by the function

$$V(t) = 12870 - k(1.1)^t, t \geq 0$$

where t is the number of years since the car was purchased and k is a constant.

(a) Write down, and simplify, an expression for the car's value when Gabriella purchased it. [2]

After two years, the car's value is \$9143.20.

(b) Find the value of k . [2]

This model is defined for $0 \leq t \leq n$. At n years the car's value will be zero dollars.

(c) Find the value of n . [2]

Working:

Answers:

(a)

(b)

(c)



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20EP19

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20EP20